Pressure around a 20-degree Circular Cone at 35-degree Angle of Attack and Low Speed

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This paper reports results from comprehensive pressure measurements of a circular cone-cylinder model in a low-turbulence $3.0 \times 1.6$ m low-speed wind tunnel. The semi-apex angle of the cone is $10^\circ$. The results consist of detailed pressure distributions over nine stations on the cone at $35^\circ$ angle of attack and Reynolds number of $0.9 \times 10^6$ based on the diameter of the cone base. The tests encompassed a complete coverage of roll angles in $9^\circ$ intervals. The test conditions are chosen to be the same as those of a known test in the literature for validation purpose. Local and overall forces and moments are calculated from the measured pressures. The variation of local side-force coefficient normalized by the local diameter of the cone, with roll angle is nearly a square-wave curve. The primary state of the boundary layer is inferred from the measured pressures to study the origin of asymmetries. The association of local side force with a characteristic pressure at a given pressure station is shown independent of roll angle for various local side force and various pressure measuring stations. The conicity of the flow is examined. The complete measured pressures are documented in the Appendix for public use.

Nomenclature

\begin{align*}
C_m &= \text{pitching-moment coefficient about cone base, pitching moment}/q_\infty SD \\
C_n &= \text{yawing-moment coefficient about cone base, yawing moment}/q_\infty SD \\
C_{Nd} &= \text{local normal force coefficient, local normal force}/q_\infty d \\
C_{N0} &= \text{overall normal force coefficient, overall normal force}/q_\infty S \\
C_{Yd} &= \text{local side force coefficient, local side force}/q_\infty d \\
C_{Y0} &= \text{overall side force coefficient, overall side force}/q_\infty S \\
c_p &= \text{pressure coefficient} \\
D &= \text{base diameter of circular cone} \\
d &= \text{local diameter of circular cone} \\
L &= \text{length of circular cone} \\
M &= \text{free-stream Mach number} \\
q_\infty &= \text{free-stream dynamic pressure}
\end{align*}

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\[ Re = \text{Reynolds number based on } D \]
\[ S = \text{base area of circular cone} \]
\[ V = \text{free-stream velocity} \]
\[ x, y, z = \text{body axes, Fig. 3} \]
\[ \alpha = \text{angle of attack} \]
\[ \theta = \text{meridian angle measured from windward generator, clockwise is positive looking upstream} \]
\[ \phi = \text{roll angle about axis of symmetry, clockwise is positive looking upstream} \]

I. Introduction

Symmetric separation vortices over a slender pointed body of revolution may become asymmetric as the angle of attack is increased beyond a certain value, causing asymmetric force and moment even at symmetric flight conditions.\(^1\) Much experimental, theoretical, and computational work has been spent on the understanding, prediction, and control of the onset of force asymmetry. The basic physical mechanism of this transition, however, is not clear. At least two possible causes for the force asymmetry were suggested mainly based on experimental investigations: (1) Inviscid hydrodynamic instability of the symmetrically separated vortices (Keener and Chapman)\(^2\) and (2) asymmetric flow separation and/or asymmetric flow reattachment on each side of the body (Ericsson).\(^3\) There is at present no general agreement on the mechanism involved in the creation of the force asymmetry.

It is known that the apex portion of the pointed slender body plays a decisive role in determining the asymmetric flow over the entire body. Since the pointed nose is locally conical in shape, the flow may be regarded as locally equivalent to that about a tangent cone. The basic features of the asymmetric flow about pointed slender bodies of revolution can be displayed by studying the flows over a circular cone.

Force tests for a circular cone of 8.13° semi-apex angle were conducted by Coe et al.\(^4\) for angles of attack up to 75° in two low speed wind tunnels by a strain gage balance system. Asymmetric force and moment onset at about 16° at zero sideslip and reached a high magnitude at 35° angle of attack and reversed in direction beyond the angle of attack. The yawing moment coefficient was given for four roll orientations in 90° intervals at a Reynolds number of 0.21 \(\times\) \(10^6\) based on base diameter. Similar data were obtained for roll angles of 0°, 180°, and 270°, but a roll angle of 90° produced a reversal in the sense of the asymmetry. The magnitudes of the asymmetry and the angle of attack at which they occurred remained relatively unchanged. The large forces and moments were caused by asymmetric shedding of vortex sheets off the pointed nose as visualized by tuft and smoke techniques. The results were found to be repeatable with no noticeable hysteresis effect.

Keener et al.\(^5\) measured the forces and moments acting on a 20° cone for angles of attack up to 88° at Reynolds numbers based on base diameter ranging from 0.3 \(\times\) \(10^6\) to 4.6 \(\times\) \(10^6\) and Mach numbers ranging from 0.1 to 0.7 by a force balance in a large subsonic wind tunnel. Angle of attack was varied from 0° to 88° at zero sideslip. The cone with pointed nose experienced steady side force at high angles of attack at zero sideslip. A large side force developed at angles of attack between 20° and 75°. The side force changed from side to side as the angle of attack increased and was accompanied by dynamic oscillations. The direction and magnitude of the side force was sensitive to the body geometry near the nose. The maximum side force reached about 0.9 times the normal force. The angle of attack of onset of side force was about 20° and not strongly influenced by Reynolds number or Mach number.

An RAE unpublished force measurement of A.R.G. Mundell (1982) was reported in Ref. 6. A large side force was obtained for a 20° cone at a high angle of attack, zero sideslip and low speed by a balance, while the separation positions were not greatly asymmetric as recorded by oil flow method.

Reference 6 also reported that a series of pressure measurements of a nose cone-fairing segment-after cylinder model had been conducted in the RAE 5 m low-speed pressurized wind tunnel. The nose cone has 10° semi-apex angle. The model was rolled in 10° intervals through the entire roll range. There were six pressure-tapping stations in which four stations were located on the nose cone and the rest two stations on the fairing segment. Each station housed 36 holes equally-spaced around the circumference. Part of the data on a front station, 148.5 \(mm\) from the apex at 35° angle of attack was reproduced in Ref. 6, and used to demonstrate that the maximum side force state was associated with a characteristic circumferential pressure distributions at the foremost station of the cone, which was independent of roll angle. It was conjectured...
by Luo\textsuperscript{7} that such an association of the local side force with a characteristic pressure distribution may hold true for every local side force not limited to the maximum, and on all pressure stations not limited to the foremost station at the given angle of attack.

The water flow about a circular cone of 12° semi-apex angle inclined at 16° is seen to be sensibly conical.\textsuperscript{8} The slender conical flow of an incompressible inviscid fluid has been analyzed by many authors. Dyer et al.\textsuperscript{9} found non-unique stationary (symmetric and asymmetric) vortex-pair positions even when symmetric separation positions are postulated with respect to the incidence plane. The stability of the stationary solutions were studied by Pidd et al.\textsuperscript{10} and Cai et al.\textsuperscript{11, 12} Pidd et al. showed that the stationary conical solutions are convectively stable under certain conditions. Cai et al. found that no stationary conical solutions are stable based on the global stability analysis.

The purpose of the present paper is to study the asymmetric features which are independent of roll angle and, thus, can be applied to different models of the same geometry specifications under the same flow conditions. The present work starts with extensive pressure measurements over a circular cone at Fiddes’ test conditions, since no complete measurement data are available to the present authors. No flow field surveys were made in the present tests.

In the following sections, the experimental setup is briefly reviewed. Variation of local side-force over the entire range of roll angle is calculated and the primary boundary-layer state is inferred from the measured pressures. Association of local side force with a characteristic pressure is shown for various local side forces at various pressure stations. The conicity of the flow is studied. Asymmetric features independent of roll angle are clarified and validated with known theoretical predictions and other experimental observations. Conclusions are lastly offered. The pressure-measurement data are documented in the Appendix for public use.

II. Experimental Setup

The tests are conducted in the NF-3 wind tunnel at the Aerodynamic Design and Research National Laboratory, Northwestern Polytechnical University. The test section has a 3.0 × 1.6 m cross section, and a length of 8.0 m. The contraction ratio is 20 : 1. The free-stream turbulence level is 0.045\% for wind speeds of 20 ~ 130 m/s.

The model comprises a nose cone of 10° semi-apex angle faired to a cylindrical afterbody as shown in Fig. 1.

The tip portion of the cone from the apex to a station of 150.0 mm along the body axis is separately made to facilitate the pressure-tube installation inside the model. After assembling the two portions, the junction was filled with grease and polished. The model is made of metal and constructed to an average tolerances of ±0.05 mm with a surface finish of nearly ±0.8 μm. The fore-body including the nose cone and the fairing portion is roll-able and the after cylinder is mounted onto the model support. The junction between the fore-body and the after cylinder is carefully machined so that the surface discontinuity is less than 0.025 mm.

The fore-body’s roll orientation is facilitated by clamping the fore-body to the axis of a remotely controlled motor which is housed in the after cylinder. The model can be set at any roll angle between 0 and 351° from the chosen datum in 9° intervals. The accuracy of the roll-angle set is about 1\%. The pressure instrumentation is confined to the nose cone and is well forward of the model support. The pressure tapings are placed at 9 stations along the model’s axis as shown in Fig. 1. Stations 1 and 2 have 12 and 18 pressure orifices, respectively, and the rest stations have 36 pressure orifices. The pressure orifices in each station are equally-spaced around the circumference and arranged from the same datum for all stations. The total number of the pressure orifices is 282.

The static pressure at the each pressure orifice is transmitted by a rubber tube passing through the base of the afterbody to the pressure-measurement system outside the test section. The system consists of 24 scan-valves each of which has 16 channels and one pressure transducer of modulus 9816 of the PSI Company with an accuracy of ±0.05\%. The pressure-measurement readings for each test case were taken 115 times in 0.05 s intervals and then time-averaged. The fluctuations of the readings are small and the time-averaged data are meaningful. A thorough job of cleaning the model was done prior to each run of the wind tunnel.

Figure 2 shows the test model rigidly supported in the wind tunnel.

The accuracy of the experimental setup has been verified by showing that the measured pressure distri-
Figure 1. The cone-cylinder model.
Figure 2. The model in the wind tunnel.
butions over the cone at zero angle of attack and zero sideslip are axi-symmetric.\textsuperscript{13}

The cone-cylinder model is tested at $\alpha = 35^\circ$, $V = 80 \text{ m/s}$, $M = 0.24$ and $Re = 0.9 \times 10^6$. 40 roll orientations in $9^\circ$ intervals are tested. Readings from 17 out of the 282 pressure orifices are abnormal in the tests at certain roll orientations. The abnormality may be caused by the twisting of the bunch of the 282 pressure tubes during the fore-body roll. The corrected pressure coefficient is calculated with a linear interpolation from the neighboring normal values. There are no abnormal pressure readings on Stations 1, 3, 8. Table 1 gives the pressure orifice number which yields abnormal pressure readings on various stations. The pressure orifice is numbered in the counter clockwise direction with Orifice 1 located at $\theta = 230^\circ$ when $\phi = 0$ for all measured stations.

Table 1. Number of the pressure orifice which yields abnormal readings on various stations.

<table>
<thead>
<tr>
<th>Station</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>All roll angles</td>
<td>12</td>
<td>4, 30</td>
<td>1, 6, 11</td>
<td>29</td>
<td>3, 27</td>
<td>None</td>
</tr>
<tr>
<td>Partial roll angles</td>
<td>None</td>
<td>None</td>
<td>10</td>
<td>3, 6, 11, 14, 32, 35</td>
<td>None</td>
<td>16</td>
</tr>
</tbody>
</table>

III. Local Side- and Normal-force Coefficients versus Roll Angle

The measured pressures are integrated to give side- and normal-forces in the body axes shown in Fig. 3, where the coordinate plane $oxz$ coincides with the incidence plane. The local side- and normal-force coefficients, $C_{Yd}$ and $C_{Nd}$ are based on the local diameter $d$ rather than the base diameter $D$, in order to confer more information as shown by Hall.\textsuperscript{14}

Figure 4 presents local side- and normal-force coefficients vs. $\phi$ at $\alpha = 35^\circ$ for all stations. The local side-force coefficient varies with roll angle in a nearly square wave-form, and jumps between maximum positive and maximum negative $C_{Yd}$ for all pressure stations at about the same roll angles with an interval of 20 to 30$^\circ$. The direction of the local side force is about the same from the tip to the base of the cone at a given roll angle.

The magnitudes of the maximum positive and maximum negative local side-force coefficients for each station are nearly equal. The small deviation in magnitude may be attributed to the inaccuracies of the experimental setup. Every positive local side force is associated with a negative one of the same magnitude.
Figure 4. Local side- and normal-force coefficients vs. $\phi$ at $\alpha = 35^\circ$, Stations 1 through 9, $Re = 0.9 \times 10^6$. 
over a different roll angle interval. The local normal-force coefficient is nearly invariant over the entire roll orientations at each station. The maximum value of $C_{Yd}$ has the same order of magnitude as $C_{Nd}$ at $\alpha = 35^\circ$.

Fiddes$^6$ gave the local side force versus roll angle over a circular cone of 10° semi-apex angle under about the same flow conditions for a front station $x = 148.5\ mm$. In comparison with the present result at Station 3, $x = 166.8\ mm$, the variation patterns of $C_{Yd}$ versus roll angle are entirely different between our measurement and those by Fiddes. This deviation is attributed to the differences in the micro surface imperfections of the two models.

The variation pattern of $C_{Yd}(\phi)$ obtained for one model can have no general significance, because it pertains the particular micro surface imperfections of the model. The surface imperfections are random and unpredictable. Another model made to the the same geometry specifications would have different surface imperfections and, hence, a different roll-angle dependence.

IV. Primary State of Boundary Layer from Pressures

Although no flow field surveys were made in the present tests, the primary state of the boundary layer can be inferred from the measured pressures as shown by Hall$^{15}$ by comparing the flow field visualizations with the corresponding pressure measurements. A usual pressure distribution is dense enough to yield the primary picture of the cross flow, but not dense enough to resolve the fine (secondary) structure of the flow. The primary state of the boundary layer, in general, can illuminate the origin of the pressure asymmetry.

Figure 5(a) shows the pressure distribution at $\alpha = 35^\circ$, Station 3, and roll angle $\phi = 0$. Both $\theta$ and $\phi$ are measured from the windward generator and clockwise is positive when looking upstream. The Reynolds number based on $D$ is $0.9 \times 10^6$. At this Reynolds number, the separation is generally either transitional separation or turbulent separation.$^{16}$ Fig. 5(a) pertains to the turbulent separation, and the characteristic points are labeled.

1. Point $A$, attachment point slightly offset to port side from incidence plane at $\theta = 5^\circ$;
2. Point $P_1$, suction peak point on port surface at $\theta = 70^\circ$;
3. Point $S_1$, turbulent separation point on port surface at $\theta = 110^\circ$;
4. Point $P_{21}$, first suction peak point on starboard surface at $\theta = 260^\circ$ or $-100^\circ$;
5. Point $S_2$, turbulent separation point on starboard surface at $\theta = 220^\circ$ or $-140^\circ$; 

6. Point $P_{22}$, second suction peak point on starboard surface at $\theta = 190^\circ$ or $-170^\circ$.

The suction peaks $P_1$ and $P_{21}$ result directly from the convex shape of the circular cross-section. The second suction peak $P_{22}$ is induced by the vortex core separated from the boundary layer on the starboard surface located nearby the point $P_{22}$. No suction peak can be detected on the port side as the vortex core separated from this side is too remote from the cone. The second suction peak $P_{22}$ enhances the first suction peak $P_{21}$ and delays the separation $S_2$ on the same side of the body. The cross flow streamlines around the cone surface inferred from the measured pressure are shown in Fig. 5(b). The state of the boundary layer over both port and starboard surfaces is identified as fully turbulent separation. The asymmetric pressure distribution results in a local side force pointing to the starboard side, $C_{Yd} = 0.52$ at the roll angle, $\phi = 0$ and Station 3. It is seen that the local side force and the asymmetric separation both are attributed to the asymmetric vortex configuration.

Moreover, beneath the primary vortex core described above there appears a secondary separation vortex over the body surface as observed by Keener\textsuperscript{17} using a oil-flow technique. Unfortunately, the present pressure data are not dense enough to conclusively describe the secondary separation.

The pressure distribution and the boundary-layer state at Station 3 in Fig. 5 are typical of all rear stations. The pressure distribution and the boundary-layer state at the front stations are somewhat different from those at the rear stations. Figure 6 presents the pressure coefficient vs. $\theta$ and cross-flow streamline at $\alpha = 35^\circ$, Station 1, and $\phi = 0$. Fig. 6(a) pertains to the laminar separation, because the local Reynolds number at Station 1 is low. The characteristic points are labeled in Fig. 6(a).

1. Point $P_1$, suction peak point on port surface at $\theta = 80^\circ$;

2. Point $S_1$, laminar separation point on port surface at $\theta = 80^\circ$;

3. Point $P_{21}$, first suction peak point on starboard surface at $\theta = 290^\circ$ or $-70^\circ$;

4. Point $S_2$, laminar separation point on starboard surface at $\theta = 290^\circ$ or $-70^\circ$;

5. Point $P_{22}$, second suction peak point on starboard surface at $\theta = 230^\circ$ or $-130^\circ$.

The suction peak $P_1$ and $P_{21}$ overlap with the corresponding separation points, since the pressure data at Station 1 are not dense enough to resolve them.

![Figure 6. Pressure coefficient vs. $\theta$ and streamline in cross-flow plane at $\alpha = 35^\circ$, Station 1, $\phi = 0$.](image-url)
Table 2 summarizes the location of the characteristic points of the pressure distribution and the local side force coefficient at Stations 1 through 9, $\alpha = 35^\circ$, and $\phi = 0$.

**Table 2. Location of characteristic points at Stations 1 through 9, $\alpha = 35^\circ$, $\phi = 0$, $Re = 0.9 \times 10^6$.**

<table>
<thead>
<tr>
<th>Station</th>
<th>$P_1$</th>
<th>$S_1$</th>
<th>$P_{21}$</th>
<th>$S_2$</th>
<th>$P_{22}$</th>
<th>$C_{Y_d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80°</td>
<td>80°</td>
<td>290°/−70°</td>
<td>290°/−70°</td>
<td>230°/−130°</td>
<td>0.72</td>
</tr>
<tr>
<td>2</td>
<td>90°</td>
<td>90°</td>
<td>290°/−70°</td>
<td>290°/−70°</td>
<td>210°/−150°</td>
<td>0.76</td>
</tr>
<tr>
<td>3</td>
<td>70°</td>
<td>110°</td>
<td>260°/−100°</td>
<td>220°/−140°</td>
<td>190°/−170°</td>
<td>0.52</td>
</tr>
<tr>
<td>4</td>
<td>70°</td>
<td>100°</td>
<td>250°/−110°</td>
<td>220°/−140°</td>
<td>180°/−180°</td>
<td>0.50</td>
</tr>
<tr>
<td>5</td>
<td>80°</td>
<td>100°</td>
<td>250°/−110°</td>
<td>210°/−150°</td>
<td>170°/−190°</td>
<td>0.49</td>
</tr>
<tr>
<td>6</td>
<td>80°</td>
<td>100°</td>
<td>260°/−100°</td>
<td>220°/−140°</td>
<td>170°/−190°</td>
<td>0.47</td>
</tr>
<tr>
<td>7</td>
<td>80°</td>
<td>100°</td>
<td>260°/−100°</td>
<td>210°/−150°</td>
<td>170°/−190°</td>
<td>0.45</td>
</tr>
<tr>
<td>8</td>
<td>80°</td>
<td>110°</td>
<td>250°/−110°</td>
<td>210°/−150°</td>
<td>160°/−200°</td>
<td>0.39</td>
</tr>
<tr>
<td>9</td>
<td>80°</td>
<td>110°</td>
<td>250°/−110°</td>
<td>210°/−150°</td>
<td>160°/−200°</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Figure 7 presents the sketches of the characteristic lines of the pressure distribution on the cone surface as if it were unwrapped at $\alpha = 35^\circ$ and $\phi = 0$. From Station 1 to Station 2, the separations on both the starboard and port sides are found to be on the windward side of the cone and laminar. They move to the leeward side of the cone and become turbulent starting from Station 3. The positions of the second suction peak, $P_{22}$ indicate that a nearby vortex core separated from the starboard boundary layer appears at Station 1, extends inboard, and crosses the incidence plane between Stations 4 and 5 to the port side. The inboard deflection of the vortex core is induced by the vortex separated from the port boundary layer and positioned above the starboard vortex. The data in the Appendix and Fig. 4 show a general decline of the strength of the second suction peak and the side force coefficient from Station 1 to Station 9. The local side-force coefficient $C_{Y_d} = 0.72$ at Station 1 and decreases to $C_{Y_d} = 0.39$ at Station 9. This may be caused by the diffusion of the vortex core.

It is seen that the large lateral pressure difference over the cone is induced by the asymmetric vortex pair separated from the cone. It indicates that the mechanism for generating large side force is the interaction between the separation vortices and the boundary conditions on the cone surface. The interaction results in multiple vortex configurations. The stable one occurs with surface imperfections at the body tip acting as the instability trigger. At some roll angles, the geometric perturbations are more effective than other roll angles. The nearly bistable asymmetry is established by the dominant features of the tip geometry. It is noted that the boundary conditions on the model are independent of the micro surface imperfections, and so are the vortex configurations. Large side force occurs even when the separation positions are not greatly asymmetric. The mechanism for the asymmetries is identified as the hydrodynamic instability of the separated vortices. How the micro surface imperfections actually act as the instability trigger remains unclear.

V. **Association of Local Side Force with Characteristic Pressure Distribution**

Fiddes discovered from his pressure measurements that the maximum local side force at the front cross-section of the cone model is associated with a characteristic pressure distribution which is independent of the roll angle. Whether this is true for local side forces other than the maximum and at other cross-sections along the cone model are investigated in this section.

Station 3 is first studied. Figures 8 (a) and (b) compare the pressures corresponding to maximum positive and negative $C_{Y_d}$ (at different $\phi$), respectively. Figures 8 (c) and (d) compare the pressures corresponding to two nearly-equal intermediate $C_{Y_d}$. The values of $C_{Y_d}$ and $\phi$ are shown in the figure. The two pressures giving the same $C_{Y_d}$ almost coincide in each case. A small horizontal shift between the two pressure distributions in Fig. 8 (d) may be caused by the inaccuracy in the roll-angle setup. The slight differences of the supposedly-equal $C_{Y_d}$ may also contribute to the small deviations.

Figure 9 compare the pressures corresponding to equal-magnitude, but opposite-sign local side forces, where the pressure giving negative $C_{Y_d}$ is plotted against the complementary angle of $\phi$, $(360^\circ − \theta)$. A small horizontal-shift deviation prevails over the two cases, which is attributed to the inaccuracy of the roll-angle
Figure 7. Sketches of characteristic lines of the pressure distribution on the unwrapped cone surface at $\alpha = 35^\circ$, $\phi = 0$, $Re = 0.9 \times 10^6$. 
Figure 8. Comparison of pressures corresponding to equal (or nearly equal) local side forces at different \( \phi \), Station 3, \( \alpha = 35^\circ \).
setup brought about by the change of hand in one of the two pressures. Otherwise, the two pressures are essentially the same in each case.

![Figure 9. Comparison of pressures corresponding to equal-magnitude, opposite-sign local side forces at different \( \phi \), Station 3, \( \alpha = 35^\circ \).](image)

For the foremost Station 1, Figure 10 (a) and (b) compare the pressures corresponding to maximum positive and negative \( C_{Y,d} \), respectively. The two pressures agree with each other in each case.

![Figure 10. Comparison of pressures corresponding to equal local side forces at different \( \phi \), Station 1, \( \alpha = 35^\circ \).](image)

For Station 5, Figure 11 (a) compares the pressures corresponding to maximum negative \( C_{Y,d} \) and (b) compares those giving equal-magnitude, but opposite-sign \( C_{Y,d} \). The deviations in (b) are due to the reasons stated above. Otherwise, the two pressures essentially agree in each case.

Figure 12 (a) compares the pressures corresponding to equal-magnitude, but opposite-sign \( C_{Y,d} \) and Station 7, and (b) compares the pressures corresponding to maximum negative \( C_{Y,d} \) at Station 9. The agreements are similar as above cases.

In summary, for every local side force, not limited to the maximum, there exists a characteristic pressure distribution, no matter at what roll angle it occurs. And this is true for all stations along the cone length.
Figure 11. Comparison of pressures corresponding to equal-magnitude local side forces at different $\phi$, Station 5, $\alpha = 35^\circ$.

Figure 12. Comparison of pressures corresponding to equal-magnitude local side forces at different $\phi$, Stations 7, 9, $\alpha = 35^\circ$. 
Therefore, every local side force is associated with a characteristic pressure which is independent of roll angle, i.e., independent of micro surface imperfections of the model. The association may be applied to circular cones of the same apex angle under the same test conditions. This is verified by a comparison with Fiddes’ results\(^6\) in the next section.

VI. Validation of \(C_{Y_d} - C_p(\theta)\) Association with Fiddes’s

Figure 13 compares the present characteristic pressure with that obtained by Fiddes\(^6\) under about the same test conditions. Both models are of 10° semi-apex angle and tested at \(\alpha = 35^\circ\), \(Re = 0.9 \times 10^6\) and low speeds. The positions of the pressure-measuring station are close to each other as shown in the figure. Both pressures give the maximum local side force. To match with the pressures given by Fiddes, the roll angle of the present model is chosen at 342°.

![Figure 13. Pressure distribution at Station 3, \(\phi = 342^\circ\) compared with Fiddes’ result\(^6\) at \(\alpha = 35^\circ\), \(Re = 0.9 \times 10^6\).](image)

A deviation occurs in the state of the boundary layer. The boundary layer on Fiddes’ model is laminar at the pressure-measuring station as recorded by a china-clay technique.\(^6\) The primary state of boundary layer is also inferred from the measured pressures.\(^15\) Fiddes’ pressure distribution indicates a laminar separation on the starboard side and a transitional separation on the port side at the pressure station. The present pressure distribution indicates a turbulent separation on both sides at the pressure station. The differences of the boundary-layer state are attributed to the different micro surface imperfections of the two models and the different free-stream turbulence levels of the two wind tunnels. Table 3 compares \(\theta\) of the characteristic points of the two pressure distributions, where \(R_1\) denotes the turbulent re-attachment point of the transitional separation.

**Table 3. Characteristic points of pressures at Station 3, \(\phi = 342^\circ\) compared with Fiddes’ result\(^6\) at \(\alpha = 35^\circ\), \(Re = 0.9 \times 10^6\).**

<table>
<thead>
<tr>
<th>Point</th>
<th>(P_1)</th>
<th>(S_{11})</th>
<th>(R_1)</th>
<th>(S_{12})</th>
<th>(P_{12})</th>
<th>(P_2)</th>
<th>(S_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present</td>
<td>93°</td>
<td>133°</td>
<td>N/A</td>
<td>N/A</td>
<td>163°</td>
<td>-87°</td>
<td>-117°</td>
</tr>
<tr>
<td>Fiddes(^6)</td>
<td>90°</td>
<td>110°</td>
<td>130°</td>
<td>140°</td>
<td>160°</td>
<td>-70°</td>
<td>-90°</td>
</tr>
</tbody>
</table>
Figure 14 gives the present streamline in the cross-flow plane at Station 3 and $\phi = 342$ compared with Fiddes’ result\(^6\) at $\alpha = 35^\circ$ and $Re = 0.9 \times 10^6$.

Although the separation states are greatly different between the two models, the positions of the second suction peak or the nearby separated vortex are close to each other ($\theta \approx 160^\circ$), and the pressure distributions are essentially identical. The agreement of the two pressures as shown in Fig. 13 clearly demonstrates that the pressure distribution associated with a given local side force is independent of the micro surface imperfections of the model, and the maximum local side force at a given station is also independent of the model’s imperfections.

It is noted that the maximum $C_{Yd}$ given by Ref. 6 is 1.2, while the present result is about 0.6. The deviation may be due to different normalizations used for $C_{Yd}$.

VII. Pressure and Side- and Normal-Force Distributions along the Cone Axis

Figure 7 has shown that all the pressure characteristic lines are not conical rays. In this subsection, the pressure and local-force distributions along the cone center-line is studied. Figure 15 presents $c_p$ versus $x/L$ at constant values of $\theta$ for $\phi = 0$ and $171^\circ$. It is seen that the pressure is not constant at a given $\theta$ along the cone length especially in the tip region.

The longitudinal distributions of the local side- and normal-force coefficients are investigated. Fig. 16 gives $C_{Yd}$ and $C_{N0}$ versus $x/L$ at various roll angles of $90^\circ$ intervals. It is seen that $C_{Yd}$ and $C_{N0}$ are not constant along the cone axis at any given roll orientation. It is noted that $C_{Yd}$ would approach a non-zero value when $x$ approaches zero. This indicates that the pressure asymmetry starts right at the apex of the body, as the local side force coefficient $C_{Yd}$ is normalized by the local diameter $d$. This belief is supported by the fact that the boundary-layer thickness at the apex is so thin that the surface imperfections can penetrate.

Both distributions of the pressure and the local side- and normal-forces indicate that the flow about the circular cone at the angle of attack of $35^\circ$ is non-conical. This result agrees with the theoretical prediction that no conical (symmetric or asymmetric) vortex pair is stable over slender circular cone at high angles of attack under small temporary perturbations, no matter where separation (symmetric or asymmetric) positions are postulated,\(^12\) and also with other experimental observation. Pidd et al.\(^10\) reported non-conical flows in tests of a circular cone of semi-apex angle of $10^\circ$ at $\alpha = 35^\circ$ and many roll angles in an RAE 5 m low-speed wind tunnel.

VIII. Overall Forces and Moments

The overall forces and moments over the cone are calculated from integrating the measured pressures at the nine stations. The longitudinal length of the cone from the apex to the base is divided into nine elements by the eight mid points of the neighboring stations. On each element, the pressure is assumed to be constant along each meridian line. The overall side- and normal-force coefficient, $C_{Y0}$ and $C_{N0}$ are defined by the base area of the cone $S$. The overall yawing- and pitching-moment coefficients, $C_n$ and $C_m$ are defined by $S$ and the base diameter, $D$. The moment center is positioned at the base of the cone. The overall force and moment coefficients versus roll angle are presented in Fig. 17. The correlations of $C_{Y0}$ and $C_n$ with $\phi$ have the same fashion of those for $C_{Yd}$.

A strain-gage measurement for a $20^\circ$ cone was performed by Keener et al.\(^5\) at one roll orientation, $\alpha = 35^\circ$, $Re = 10^6$ and free-stream Mach number of 0.25. Their results happen to coincide with the present at $\phi = 243^\circ$ as shown in Fig. 17.

From Fig. 17, the limiting values of the overall force and moment coefficients are obtained as shown in Table 4. The overall side force and yawing moment measured by Keener et al. lie between the limiting values of Table 4. It confirms that the limiting values can be applied to cones of same apex angle under the same flow conditions.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$C_{Y0}$</th>
<th>$C_{N0}$</th>
<th>$C_n$</th>
<th>$C_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limiting value</td>
<td>$\pm 0.75$</td>
<td>1.10</td>
<td>$\pm 0.85$</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Overall forces and moments obtained at other flow conditions using the same experimental setup are
Figure 14. Comparison of present streamline in cross-flow plane at Station 3, \( \phi = 342 \) with Fiddes' result\(^6\) at \( \alpha = 35^\circ \), \( Re = 0.9 \times 10^6 \).
Figure 15. Pressure coefficient vs. $x/L$ at constant $\theta$, $\phi = 0, 171^\circ$, $\alpha = 35^\circ$.

Figure 16. Local side- and normal-force coefficients vs. $x/L$ at $\phi = 0, 90^\circ, 180^\circ$ and $270^\circ$, $\alpha = 35^\circ$. 
Figure 17. Overall forces and moments coefficients vs. roll angle at $Re = 0.9 \times 10^6$ compared with those measured by Keener et al.\(^5\) at $Re = 10^6$, at $\alpha = 35^\circ$.

verified in an accompanying paper, AIAA Paper 2007-4118.\(^{13}\)

IX. Conclusions

Pressure measurements at 9 stations over a 20\(^\circ\) cone at 35\(^\circ\) angle of attack, zero sideslip and all roll angles in 9\(^\circ\) intervals are performed in a low-turbulence, low-speed, large-scale wind tunnel with a rigid support. The test flow conditions are Reynolds number based on the cone-base diameter of $0.9 \times 10^6$ and Mach number of 0.24.

At the angle of attack of 35\(^\circ\), the local side force coefficient normalized by the local diameter varies with roll angle nearly as a square-wave curve for the 20\(^\circ\) cone. Every positive local side force is associated with a negative local side force with the same magnitude at different roll angles on any given station. The local side force switches between the maximum positive and maximum negative values back and forth at about the same roll angles for all pressure stations. The maximum local side force at a give station and the maximum overall side-force and yawing-moment over all roll angles are unique. The local normal force at any given station is practically invariable with roll angle. The asymmetric pressure starts from the apex of the cone and persists down the body. Large side force occurs even when the separation positions are not greatly asymmetric. Essentially the same pressure distribution appears even when the separation states are significantly different. The separated flow is not conical over the cone.

The asymmetry onset is related to the appearance of a second suction peak in the circumferential pressure distribution. The second suction peak is caused by a nearby separated vortex. It enhances the first suction peak and delays the separation on the same side of the body. The pressure asymmetry over the body corresponds to an asymmetric vortex pair in the leeward side of the body. The experimental results show that there exist mainly two possible separated vortex configurations at the angle of attack and zero sideslip. The vortex configurations result from the interaction between the separated vortex pair and the boundary conditions on the surface of the cone. The asymmetric vortex configuration occurs in pair as mirror images as the body is axi-symmetric and inclined at zero sideslip.

The variation of side force with roll angle of the cone indicates that a stable vortex configuration occurs under the perturbations of micro surface imperfections of the model. The micro surface imperfections of the body trigger the occurrence of the stable vortex configuration. However, they cannot affect the vortex configuration, since the possible vortex configurations are independent of the micro surface imperfections of the model. The nearly square side-force variation is established by the dominant features of the surface imperfections. How the micro surface imperfections of the model actually act as the instability trigger is still
not quite clear.

The circumferential pressure distribution corresponding to the same local side force on a given station is unique. Every local side force (not limited to the maximum) is associated with a characteristic pressure correlation between local side force and the associated pressure further demonstrates that the asymmetries distribution at any given station (not limited to the foremost), which is independent of roll angle. The correlation between local side force and the associated pressure further demonstrates that the asymmetries are caused by the asymmetric vortex pair rather than asymmetric or different boundary-layer states.

Appendix

The pressure coefficients $c_p$ versus meridian angle $\theta$ at angle of attack $\alpha = 35^\circ$, $V = 80 \text{ m/s}$, $Re = 0.9 \times 10^6$, Stations 1 through 9 and all roll angles in $9^\circ$ intervals, are presented in the following FIGURES and PAGES.

FIGURE 1. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 80 \text{ m/s}$, Station 1, PAGES 1-5.
FIGURE 2. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 80 \text{ m/s}$, Station 2, PAGES 6-10.
FIGURE 3. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 80 \text{ m/s}$, Station 3, PAGES 11-15.
FIGURE 4. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 80 \text{ m/s}$, Station 4, PAGES 16-20.
FIGURE 5. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 80 \text{ m/s}$, Station 5, PAGES 21-25.
FIGURE 6. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 80 \text{ m/s}$, Station 6, PAGES 26-30.
FIGURE 7. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 80 \text{ m/s}$, Station 7, PAGES 31-35.
FIGURE 8. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 80 \text{ m/s}$, Station 8, PAGES 36-40.
FIGURE 9. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 80 \text{ m/s}$, Station 9, PAGES 41-45.

Acknowledgments

The second author wishes to acknowledge support by the Doctorate Innovation Foundation of Northwestern Polytechnical University through Grant CX200501. The authors would like to express their gratitude to Yongwei Gao, Zhongxiang Xi, Ye Yang, Zenghong Xi, Chunsheng Xiao, and Jiangnan Hao in Northwestern Polytechnical University for their valuable technical guidance and support in the wind-tunnel tests. The authors are grateful to Professor Xueying Deng of Beijing University of Aeronautics and Astronautics for the use of their pressure measurement system.

References


FIGURE 1. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 80$ m/s, Station 1
FIGURE 1. (Continued)
FIGURE 1. (Continued)
FIGURE 1. (Continued)
FIGURE 1. (Concluded)
FIGURE 2. $c_p$ vs. $\theta$ at $\alpha=35^\circ$, $V=80$ m/s, Station 2
FIGURE 2. (Continued)
FIGURE 2. (Concluded)
FIGURE 3. $c_p$ vs. $\theta$ at $\alpha=35^\circ$, $V=80$ m/s, Station 3
FIGURE 3. (Continued)
FIGURE 3. (Continued)
FIGURE 4. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V = 80$ m/s, Station 4
FIGURE 4. (Continued)
FIGURE 4. (Continued)

PAGE 18
FIGURE 4. (Concluded)
FIGURE 5. $c_p$ vs. $\theta$ at $\alpha = 35^\circ$, $V=80$ m/s, Station 5
FIGURE 5. (Continued)
FIGURE 5. (Continued)
FIGURE 5. (Continued)
FIGURE 5. (Concluded)
FIGURE 6. $c_p$ vs. $\theta$ at $\alpha=35^\circ$, $V=80$ m/s, Station 6
FIGURE 6. (Continued)
FIGURE 6. (Continued)
FIGURE 6. (Concluded)
FIGURE 7. $c_p$ vs. $\theta$ at $\alpha = 35\degree$, $V = 80$ m/s, Station 7
FIGURE 7. (Continued)
FIGURE 7. (Continued)
FIGURE 7. (Continued)
FIGURE 7. (Concluded)
FIGURE 8. $c_p$ vs. $\theta$ at $\alpha=35^\circ$, $V=80$ m/s, Station 8
FIGURE 8. (Continued)
FIGURE 8. (Continued)
FIGURE 8. (Continued)
FIGURE 8. (Concluded)
FIGURE 9. $c_p$ vs. $\theta$ at $\alpha=35^\circ$, $V=80$ m/s, Station 9
FIGURE 9. (Continued)
FIGURE 9. (Continued)
FIGURE 9. (Continued)